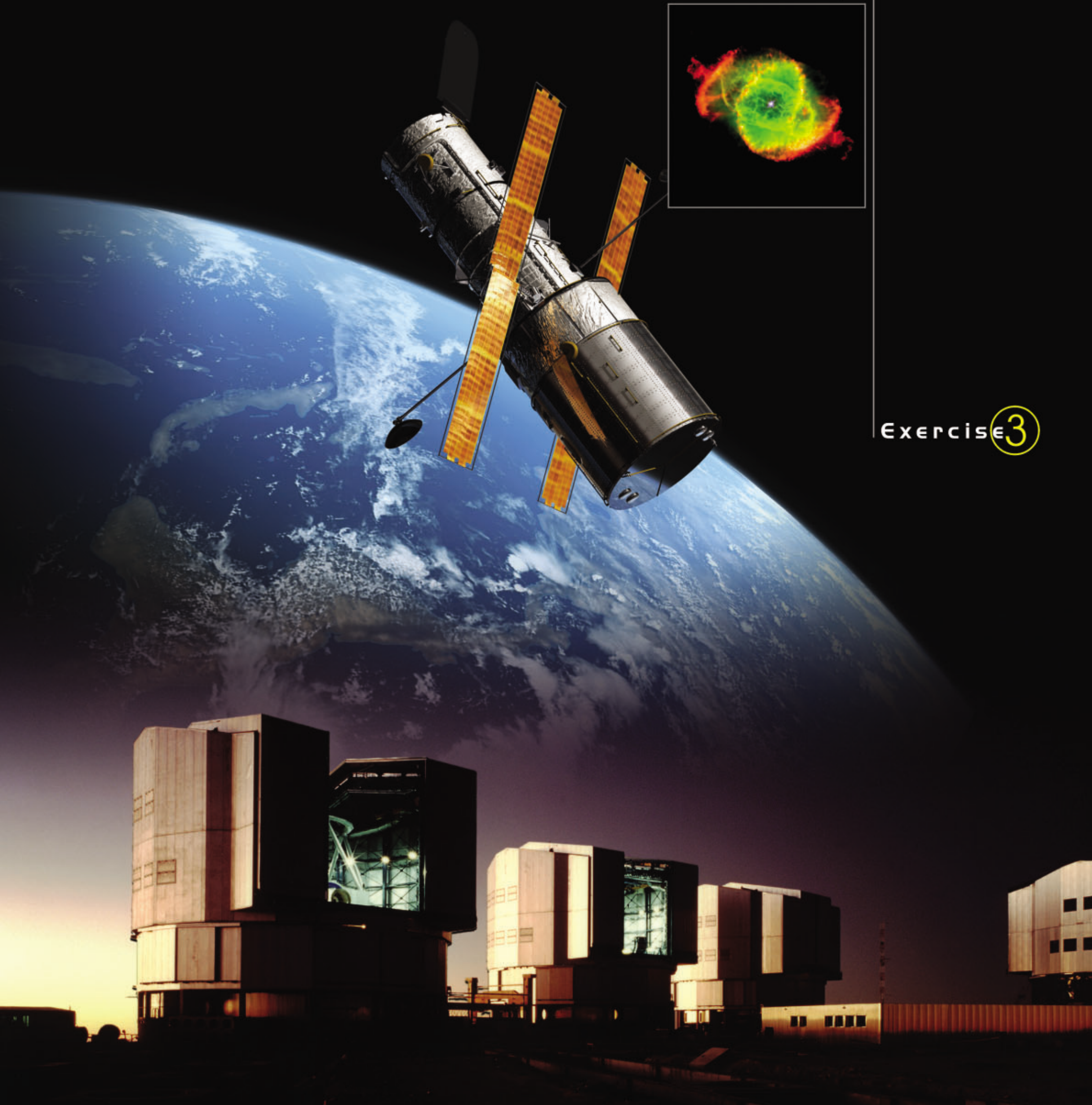
THE ESA/ESO ASTRONOMY EXERCISE SERIES

Student exercises in astronomy using observations from the NASA/ESA Hubble Space Telescope and the ESO telescopes



Exercise 3

Measuring the Distance to the Cat's Eye Nebula

Based on Observations with the NASA/ESA Hubble Space Telescope

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Measuring the Distance to the Cat's Eye Nebula

Astronomy is an accessible and visual science, making it ideal for educational purposes. Over the last few years the NASA/ESA Hubble Space Telescope and the ESO telescopes at the La Silla and Paranal Observatories in Chile have presented ever deeper and more spectacular views of the Universe. However, Hubble and the ESO telescopes have not just provided stunning new images, they are also invaluable tools for astronomers. The telescopes have excellent spatial/angular resolution (image sharpness) and allow astronomers to peer further out into the Universe than ever before and answer long-standing unsolved questions.

The analysis of such observations, while often highly sophisticated in detail, is at times sufficiently simple in principle to give secondary-level students the opportunity to repeat it for themselves.

This series of exercises has been produced by the European partner in the Hubble project, ESA (the European Space Agency), which has access to 15% of the observing time with Hubble, together with ESO (the European Southern Observatory).



Figure 1: The NASA/ESA Hubble Space Telescope
The NASA/ESA Hubble Space Telescope has presented spectacular views of the Universe from its orbit above the Earth.



Introduction

Late phases in the lives of low-mass stars

The Cat's Eye Nebula (NGC 6543) is a so-called planetary nebula. Despite the name, a planetary nebula has nothing to do with a planet. The term was introduced during the 19th century, as these objects looked rather like planets through the small telescopes of the time. Planetary nebulae form during the death throes of low-mass stars, such as the Sun, as the star's outer layers are slowly ejected.

The light emitted by most stars is a by-product of the thermonuclear fusion process known as hydrogen burning, where four hydrogen nuclei fuse into one helium nucleus.
Such fusion can only take place at the core of a star where gigantic gravitational forces push the temperatures up to about 10^7 K. At these high temperatures there is sufficient energy to overcome the electrostatic repulsive forces acting between like-charged protons and so four hydrogen nuclei (protons) can fuse to create a new nucleus, helium (see Fig. 2), and thereby release even more energy.

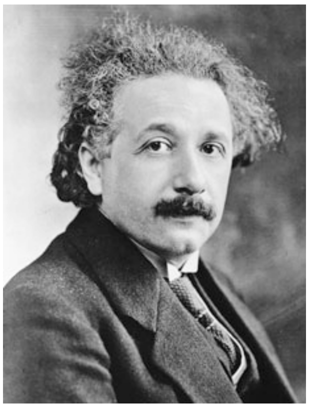


Figure 3: Albert Einstein
Einstein's famous equation $E = Mc^2$ shows the relation between mass and energy.

The mass of a helium nucleus is only 99.3% of the mass of the four original hydrogen nuclei. The fusion process converts the residual 0.7% of mass into an amount of energy — mostly light — that can be calculated from Einstein's famous equation, $E = Mc^2$. As c^2 is a large number, this means that even a small amount of matter can be converted into an awesome amount of energy. The residual 0.7% of the mass of four hydrogen nuclei involved in a single reaction may seem tiny, but when the total number of reactions involved in the fusion process is considered, there is a substantial total mass (and thus energy) involved.

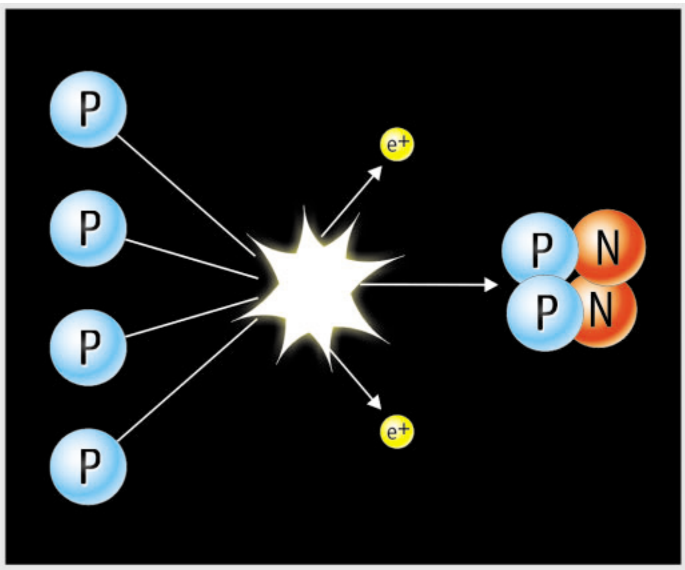


Figure 2: Hydrogen burning
The simplest mechanism for the 'generation' of energy in stars is the fusion of four hydrogen nuclei into one helium nucleus. The process has several steps, but the overall result is shown here.

The energy radiated balances the forces of gravitation, and the star remains quietly in a state of stable equilibrium for more than 90% of its life (the Sun should stay in its current stable state for another 5 billion years).

When the hydrogen supply in the core of the star is depleted and hydrogen burning is no longer possible, the gravitational forces compress the core of the star. Then the core temperature increases to 100 million K, and the helium nuclei in the core begin to fuse to form heavier elements such as carbon — the process of helium burning.
At this time the outer parts of the star swell — for a star the size of our Sun in this phase the outer envelope would extend as far as the current orbit of the Earth.

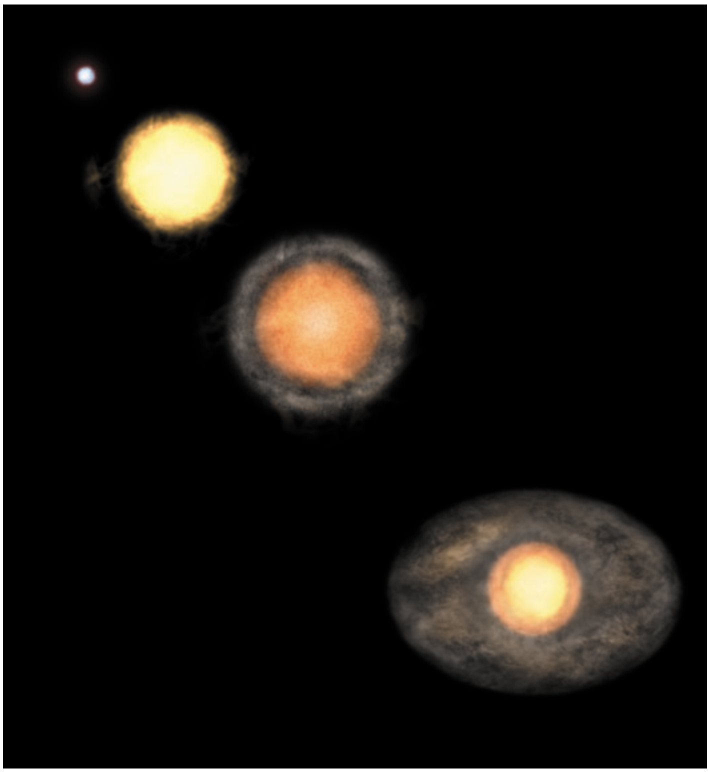


Figure 4: Late phases of a low-mass star's life
When a star reaches its final phase, it starts to burn heavier and heavier elements. At this time the star ejects dust and gas, thereby forming a planetary nebula.

Material from deep within the star is brought to the surface repeatedly during this late stage of a low-mass star's life, thereby enriching the outer envelope with elements other than hydrogen, in a process called dredge-up. The envelope is finally ejected out into space, sometimes in a spherical shell, but often in an asymmetrical shape, creating a cocoon around the dying star (see Fig. 4).

The ultraviolet light from the central core of the dying star illuminates the expelled material, highlighting the structure of the spectacular planetary nebulae we see in telescopes. Planetary nebulae are very short-lived by astronomical standards. The age of several well-known planetary nebulae — the Cat's Eye Nebula (NGC 6543) being one of them — is only around a thousand years, and they are not generally more than fifty thousand years old. After this they slowly fade into the interstellar medium, enriching it with heavy elements available for the next generation of stars.

The Sun is an ordinary low-mass star and it will most likely end its life as a spectacular planetary nebula. The Earth will not be able to sustain life when this happens, but we have about 5,000 million years before this becomes our major environmental problem.

Distances to Planetary Nebulae

In this exercise we will measure the distance to the Cat's Eye Nebula. The study of physical properties such as the size, mass, brightness and age of planetary nebulae is impossible without accurate distance measurements to the nebulae. Indeed, astronomy in general depends on accurate distance measurements.

It is not easy to measure the distances to planetary nebulae. Even though they form from so-called low-mass stars, the initial mass of the progenitor stars can still vary by as much as a factor of ten, giving individual planetary nebulae very different properties. As all planetary nebulae do not have the same size or brightness it is not possible to use such generalisations to estimate their distances. Occasionally, however,

observations can be made that allow the determination of the distance to a planetary nebula directly, as is the case with the Cat's Eye Nebula.

The Cat's Eye Planetary Nebula

The Cat's Eye Nebula lies in the constellation of Draco and is one of the most complex planetary nebulae ever seen. Images from Hubble reveal surprisingly intricate structures including concentric gas shells, jets of high-speed gas and unusual knots of gas. It is believed that the central star is actually a double star since the dynamic effects of two stars orbiting each other most easily explain the unusually complex structure of the nebula.

Analyses of the different features in the nebula, shown in Fig. 6, have been made several times before. It is known that several of the most prominent features have a different age from the central part of the nebula. The measurements that we make in this exercise will not focus on these features, but on the minor axis of the ellipsoid called E25.

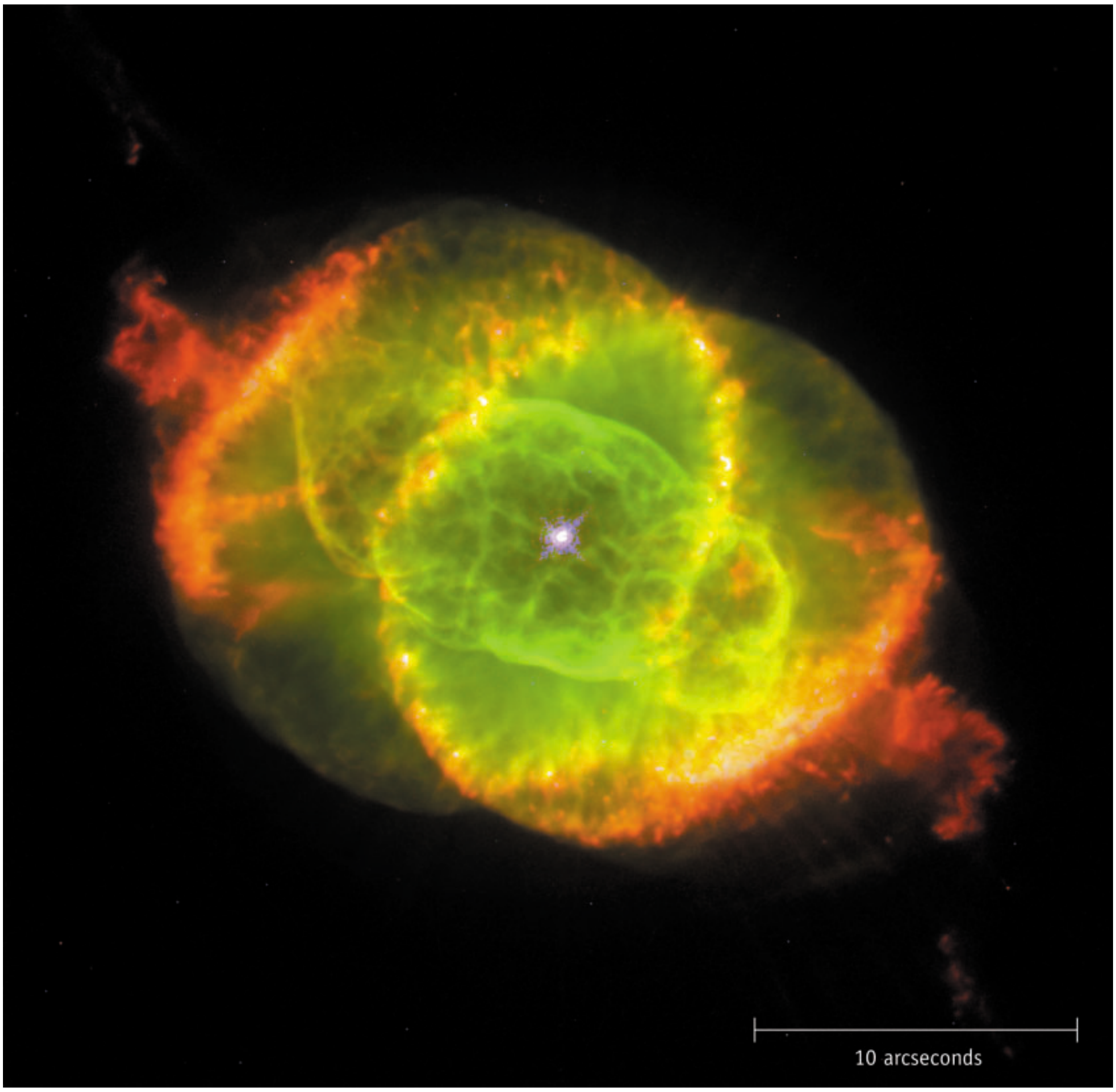


Figure 5: The Cat's Eye Planetary Nebula

This colour picture of the Cat's Eye nebula, NGC 6543, taken with Hubble's Wide Field Planetary Camera 2, is a composite of three images taken at different wavelengths. Ionised nitrogen (658.4 nm) is shown as red, double ionised oxygen (500.7 nm) is shown as green, and neutral oxygen (630.0 nm) is shown as blue. The scale of the image is indicated. The feature called E25 is the ellipsoid nearest the central star.

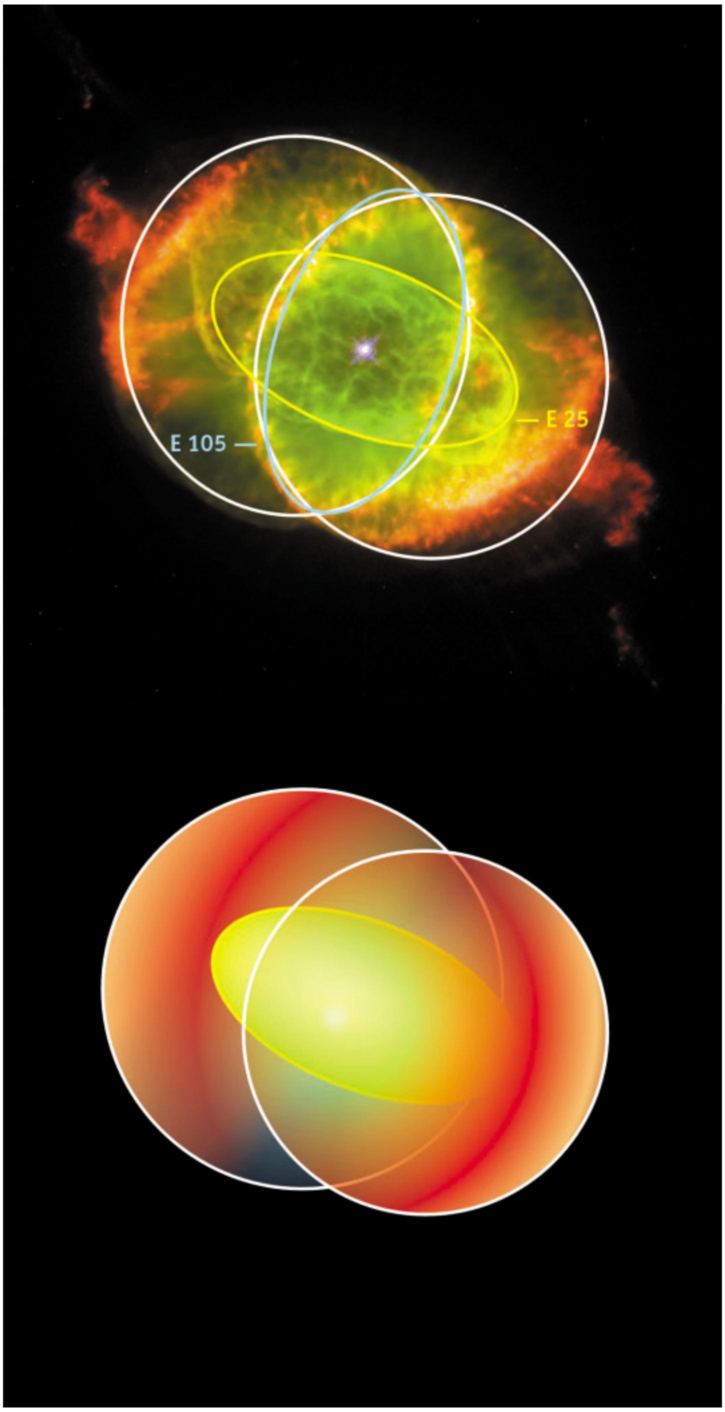


Figure 6: Geometric 3D model of the Cat's Eye Nebula
The general bipolar structure of the nebula is illustrated here. The inner ellipsoid, named E25, is marked in yellow. Adapted from Reed et al. (1999).



In the following two tasks, t is the elapsed time between the two observations.

Task 1

? Find a relation between the angular displacement, a, the time, t, and the angular velocity of the displacement, ω.

The angular velocity is measured in units of angle per unit time. It is important that this angle is measured in radians.

Task 2

? Find a relation between the linear displacement, l, the time, t, and the velocity in the direction of the linear displacement, v_t. This velocity is called the tangential velocity.

The tangential velocity is measured in km/s.

Task 3

Using the small-angle approximation in the Mathematical Toolkit, we find a relation between the distance, D, the linear displacement, l, and the angular displacement, a.

$$D = l/a$$

? Use this equation and find a relation between the distance, D, the tangential velocity, v_t, and the angular velocity, ω.

The Cat's Eye Nebula was imaged twice with Hubble, first on September 18th 1994 and then again on August 17th 1997. If the two images are displayed alternately on a computer screen in quick succession (a technique known as 'blinking'), it can be seen that the Cat's Eye Nebula has expanded in the interval between the two images. This angular expansion is not so large that it is possible to see any difference between two printed images with the eye alone, but it is still possible to determine the amount of the expansion in units of angle per unit time — as you will see for yourself in a moment.

The use of this effect, known as expansion parallax, is not all that uncommon in astronomy. It is more usually applied to images taken by radio telescopes, but here Hubble's high resolution makes it possible to determine the expansion parallax of different features in this relatively distant nebula at visible wavelengths. This makes a detailed description of the nebula possible.

Measuring the expansion parallax along the minor axis of E25 corresponds to determining an angular velocity, ω, perpendicular to the line of sight. One more piece of information is needed to calculate the distance to the nebula: the tangential velocity along the minor axis of E25. Fortunately this velocity has been obtained by a group of astronomers (Miranda & Solf, 1992) who combined the spectroscopic methods[1] with a kinetic model of the expansion of the nebula. The group concluded that the tangential velocity along the minor axis of E25 is 16.4 km/s, which corresponds to approximately 60,000 km/h.

As the tangential velocity, v_t, is given, we only need to determine the angular velocity. You will use two different methods to do this, the magnification method and the radial fitting method.

[1]Spectroscopic measurements split light into its different colours or wavelengths (e.g. with a prism). The light can then be investigated to look for a possible Doppler-shift induced by the motion of the source, and the corresponding radial velocity (towards or away from us) can be deduced. In this particular case the knowledge of the radial velocity has been combined with models of the overall expansion motions (in all directions) to deduce the tangential velocity.

The magnification method

The size of the expansion in the Hubble images is less than one pixel (picture element) and so its measurement demands a fairly sophisticated technique.
The magnification method takes the 1994 image and magnifies it until it exactly matches the 1997 image. Fig. 7 shows the method of subtracting. Note that the image taken in 1994 is not magnified.
In each of the nine small images in Fig. 8 the 1994 image was magnified by a different factor, F, (this is the number in the upper right hand corners of all the nine small images) and then the 1997 image was subtracted from the magnified 1994 image.

The more similar the two images are, the less structure will appear in the residual image. We should look for the smoothest image and the magnification factor in the corner of the image is the one that best describes the expansion of the Cat's Eye nebula from 1994 to 1997. Note that since the expansion parallax is not the same for all features, we should look for the image where 'our' feature — the minor axis of E25 — disappears.

When the magnification factor, F, is determined, ω can be calculated from this expression:

$$\omega = \frac{(F-1)d}{t}$$

where t is the time elapsed between the two images were obtained, and d is the distance in radians between the central star in the nebula and the feature (in this case the minor axis of E25) being measured. ω is measured in radians per unit time.

The following tasks take you through the calculation of the different parameters in this expression. Afterwards you will be able to calculate ω and then the distance to the Cat's Eye Nebula.

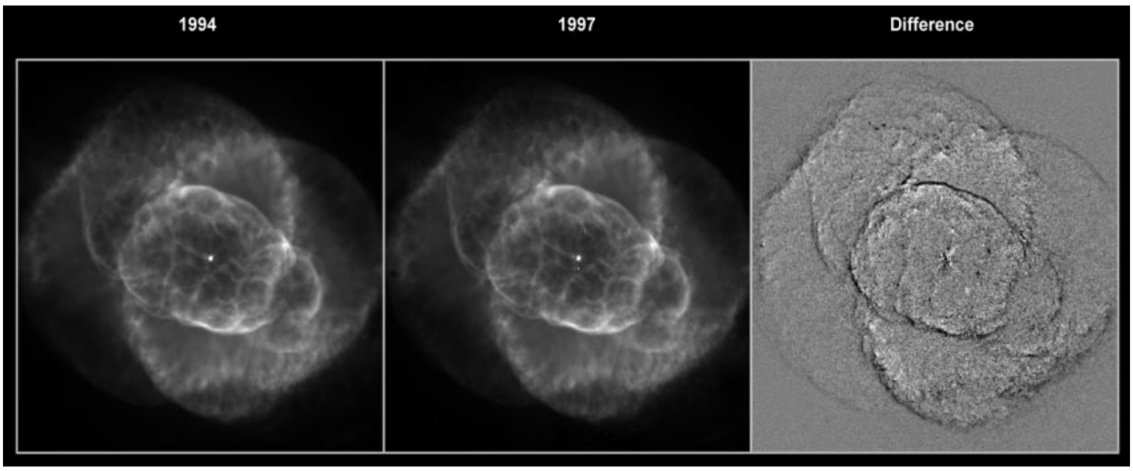


Figure 7: Highlighting the expansion requires a specific image treatment
The first image (Fig. 7a) was taken in 1994, the second (Fig. 7b) in 1997. Only an eagle-eyed observer could detect any difference between the two images without a computer. The image treatment subtracts one image from the other. The resulting image is called the residual (Fig. 7c).

Tasks

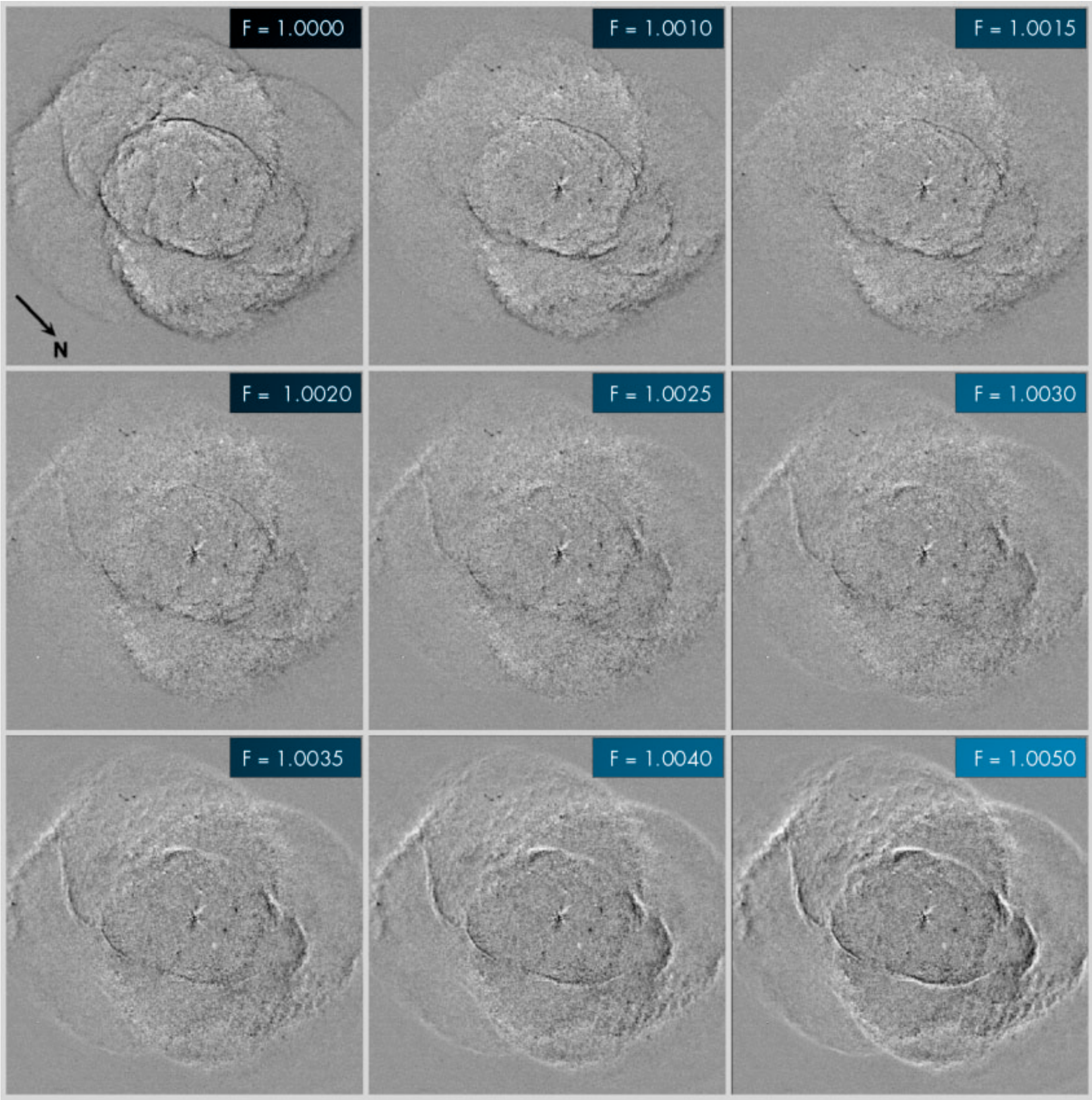


Figure 8: The nine residual images
These images are the result of magnifying the 1994 image and then subtracting the 1997 image. The magnification factor, F, is indicated on each image (from Reed et al., 1999).



Task 4

? Decide in which of the nine images in Fig. 8 the minor axis of E25 disappears or is closest to disappearing. You may decide that two images are equally good and take the average of these as the magnification factor.

Task 5

? Calculate the time between the dates when the two images were taken, and convert this period of time into seconds.
Why doesn't it matter that you don't know the exact time of day the images were taken?

Task 6

? Locate the minor axis of E25 in Fig. 5. Measure the distance, d, from the central star of the nebula to the minor axis of E25 in units of milli-arcseconds. Convert this distance to radians using the conversion factor given in the Mathematical Toolkit.

Task 7

? Now you have everything you need to calculate the expansion parallax, ω, using the magnification method.

Task 8

As mentioned before, the tangential velocity of minor axis of E25, v_t, has already been measured by a team of astronomers as 16.4 km/s.

? Calculate the distance to Cat's Eye nebula.

Before you compare your result with the one from Reed et al., you should calculate the distance to the Cat's Eye Nebula using the radial fitting method.



The radial fitting method

If we measure the pixel value of the pixels of a line passing through the central star in Fig. 5, we obtain the curve shown in Fig. 9. The peaks and valleys correspond to light and dark areas along the line and reflect the intensity of light coming from the different ridges and knots in the nebula.

The difference between curves made from the two different images from 1994 and 1997 respectively can be used to measure the expansion of the nebula.

Unfortunately the difference between the positions of features in the two images is so small (less than 1 pixel) that we cannot easily repeat this measurement here. You will have to trust the different measurements of ω from the scientists and use this to derive the distance to the Cat's Eye Nebula again. The scientists have measured ω at many different places on E25 (and also in many other points in the nebula). These measurements are indicated in Fig. 10. NB: The measurements are made in milliarcseconds/year and they have to be converted to the correct units.

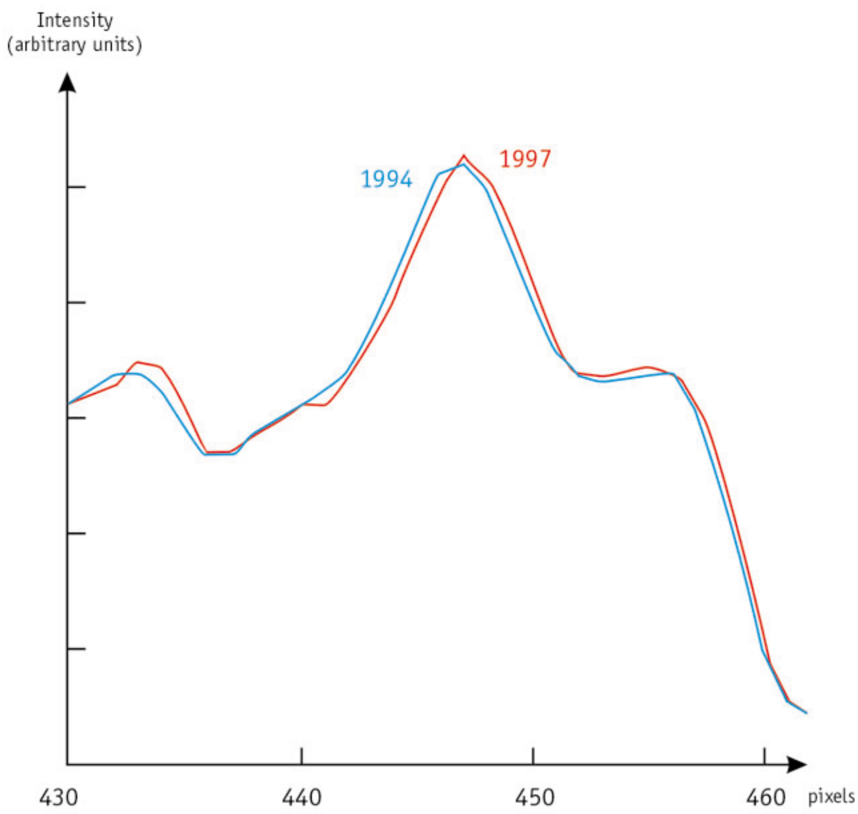


Figure 9: Intensity profiles
Two examples of how intensity measurements along a line running through the nebula look in the 1994 and 1997 images respectively. The line we have used here is the one at 12 o'clock in figure 10.

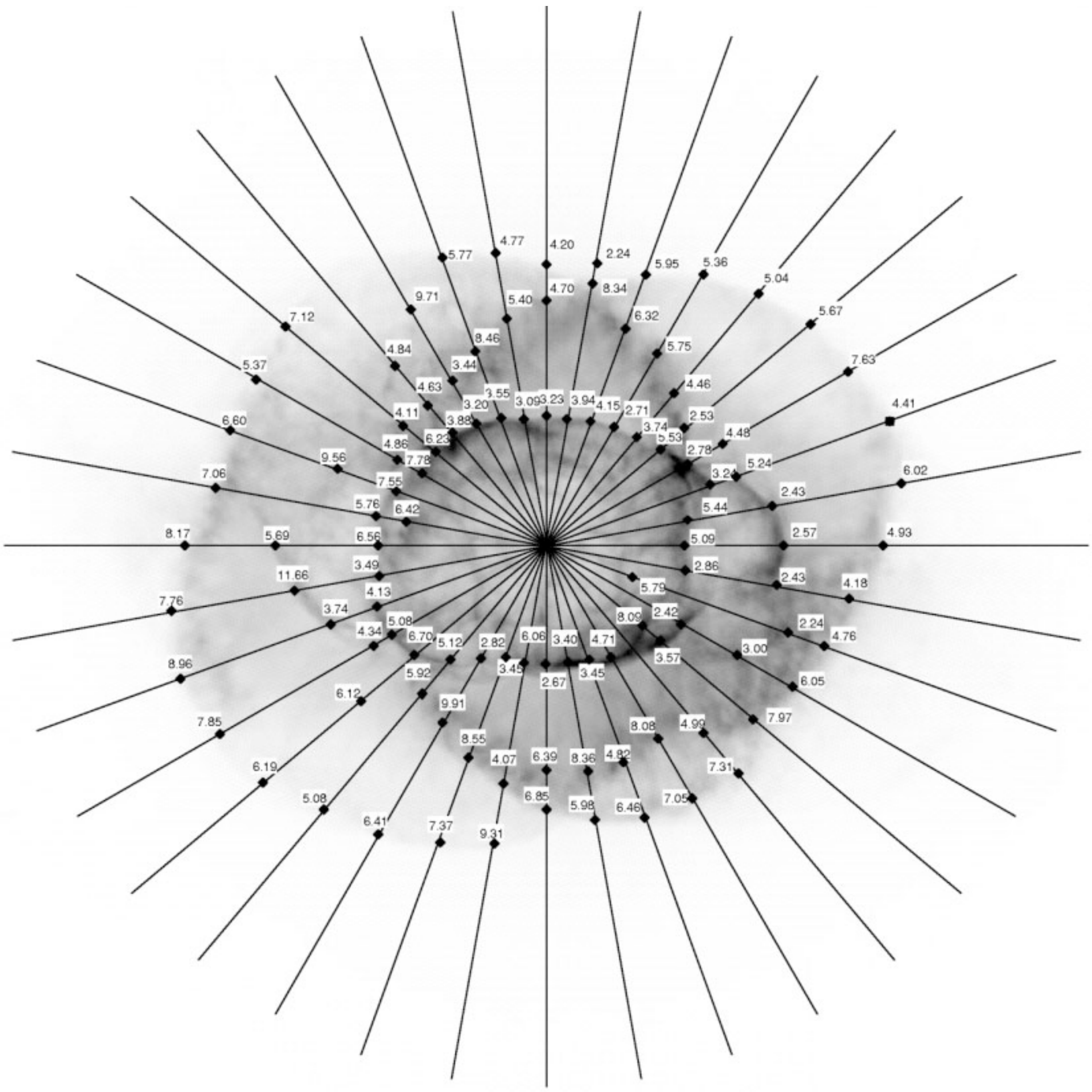


Figure 10: ω and Radial Intensity Profiles
The resulting ω's from careful fitting of radial profiles along the lines shown in the figure. ω is measured in milliarcseconds/ year (from Reed et al., 1999).



Task 9

? Identify the minor axis of E25 in Fig. 10. Read off the ω of the appropriate diamonds — take care matching the right diamonds to the right numbers!
Average the ω's you found in the figure and calculate the distance to the Cat's Eye Nebula as before.

Task 10

The kinematic age (the time that has passed since the expansion of the nebula started), T, of the inner core of the nebula can be deduced from the previous calculated values (considering the expansion rate is constant since the expansion began):

$T = d/\omega$

The value of d was found in Task 6.

? Calculate the kinematic age, T, for both values of ω you have determined.

Task 11

The result from Reed et al. for the distance to the Cat's Eye Nebula is 1001 ± 269 parsec. This result was obtained by measuring not just E25 but also by a) measuring other structures with the magnification method, b) using all the different diamonds in the radial fitting method and finally c) using a third method, called the profile method.

? Compare your result to Reed et al's result.

? Think about where you could have influenced the result significantly by the choices you have made. Redo the calculation of the distance by varying the parameters slightly. You could, for instance, choose another residual image in the magnification method or select different diamonds in the radial fitting method. Make small variations in the parameters and you will probably see a huge difference in the results.

This exercise illustrates both the difficulty of obtaining accurate distance measurements and the strength of the astronomical tools.

Further Reading

Scientific Papers

- Reed, Darren S., Balick, B., Hajian, Arsen R., Klayton, Tracy L., Giovanardi, S., Casertano, S., Panagia, N., Terzian, Y. 1999, AJ, 118, 2430–2441: *Hubble Space Telescope Measurements of the Expansion of NGC 6543: Parallax Distance and Nebular Evolution*

- Miranda, L.F., Solf, J. 1992, A&A, 260, 397–410: *Long-slit spectroscopy of the planetary nebula NGC 6543 - Collimated bipolar ejections from a precessing central source?*

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Teacher's Guide

Quick Summary

We measure the angular expansion velocity of the Cat's Eye Nebula by careful investigation of two Hubble images taken in 1994 and 1997. With the help of tangential velocity measurements from an earlier scientific paper, it is possible to determine the distance to the nebula. We also derive the distance by looking at how much the radial intensity profiles of prominent features in the two images have changed between 1994 and 1997.

In this exercise students carry out fewer measurements than in exercises 1 and 2, but are introduced to two different methods — one 'traditional' and one 'less traditional' — of calculating the distance to an astronomical object.

In the original scientific paper the astronomers use three different methods, but the third one requires very sophisticated computer programs and it is not feasible to repeat this measurement/calculation.

Task 1 and 2

Using the equation "distance = velocity · time" we find:
$\mathbf{a = \omega \cdot t}$
$\mathbf{l = v_t \cdot t}$

Task 3

Using Figure 6 in the Mathematical Toolbox, with b = l and c = D, we get:
$D = l / a = \mathbf{v_t/\omega}$

The expression of ω:
d is the angular distance to the feature in the 1994 image. F is the magnification factor. F·d is the angular distance to the feature in the 1997 image and so (F-1)d is the angular difference between the 1994 and the 1997 image. Dividing by the elapsed time we obtain the angular velocity.

Task 4

The best magnification factor is **1.00275**, as the average of 1.0025 and 1.0030.
F = 1.00275 gives the result closest to the result from the scientific paper.

Task 5

The time elapsed from 18th of September 1994 to the 17th of August 1997 (the dates are found on page 7) can be calculated easily. Note that 1996 is a leap-year
t = 3 years + 1 day – 31 days = 1065 days = $\mathbf{9.2016 \times 10^7}$ **s**

With four significant digits, plus or minus one day does not make a significant difference.

Task 6

From a 149 mm × 146 mm printed image:
44 mm corresponds to 10 arcseconds so 1 arcsecond corresponds to **4.4 mm**

A direct measurement of the distance from the central star to the minor axis of E25 yields:
17.5 mm, corresponding to d = 3.98 arcseconds = $\mathbf{1.9282 \times 10^{-5}}$ **radians** (using the conversion factor given in the box in the Toolkit).



Task 7

Calculating ω using the magnification method:

$\omega = (F-1) \times d/t = (1.00275-1) \times 1.9282 \times 10^{-5}/(9.2016 \times 10^{7}) =$ $\mathbf{5.7628 \times 10^{-16}}$ **radians/s**

Task 8

Hence the distance becomes:

$D = v_t/\omega = 16.4/(5.7628 \times 10^{-16}) = 2.8459 \times 10^{16}$ km = **922 pc**

Task 9

Calculating ω and the distance D to the Cat's Eye Nebula using the radial fitting method.
Unfortunately there is some freedom in choosing the points to measure here – and thus room for directing the result in any desired direction.

An average over 12 measuring points (at the top and bottom of the ellipsoid) gives:

top	**3.55**	**3.09**	**3.23**	**3.94**	**4.15**	**2.71**
bottom	**2.82**	**3.45**	**6.06**	**2.67**	**3.40**	**3.45**
ω_average (mas/year)	3.54					
ω_average (rad/s)	5.45×10^{-16}					
D (km)	3.01×10^{16}					
D (parsec)	976					

$\omega = 3.54$ mas/year $= 3.54 \times 10^{-3} \times 4.8481 \times 10^{-6} / (365 \times 24 \times 3600)$ radians/s =
$\mathbf{5.45 \times 10^{-16}}$ **radians/s**

Task 10

$T = d/\omega = (1.9282 \times 10^{-5})/(5.7628 \times 10^{-16}) = 3.3460 \times 10^{10}$ s = **1061 years**

with the value of ω produced by the radial fitting method:
$T = 3.539 \times 10^{10}$ s = **1123 years**

Task 11

Reed et al's result is D = 1001 ± 269 pc, T = 1039 ± 259 year.
Notice that both methods leave quite a bit of room for unconscious adjustment. It might be a good idea to let the students carry out a more formal min/max analysis. Even though there are many decisions to be taken during the measurements and calculation, it is still not possible to get totally unreasonable results.

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